Centroids and Centers of Gravity

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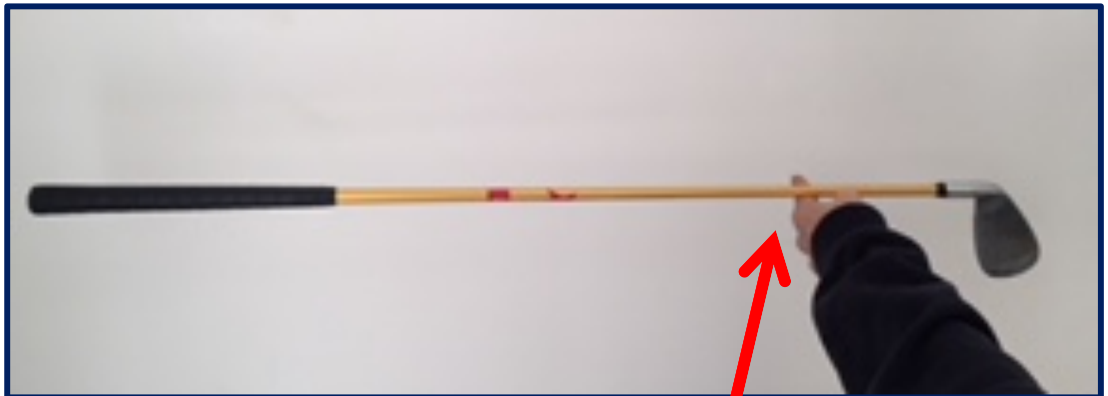
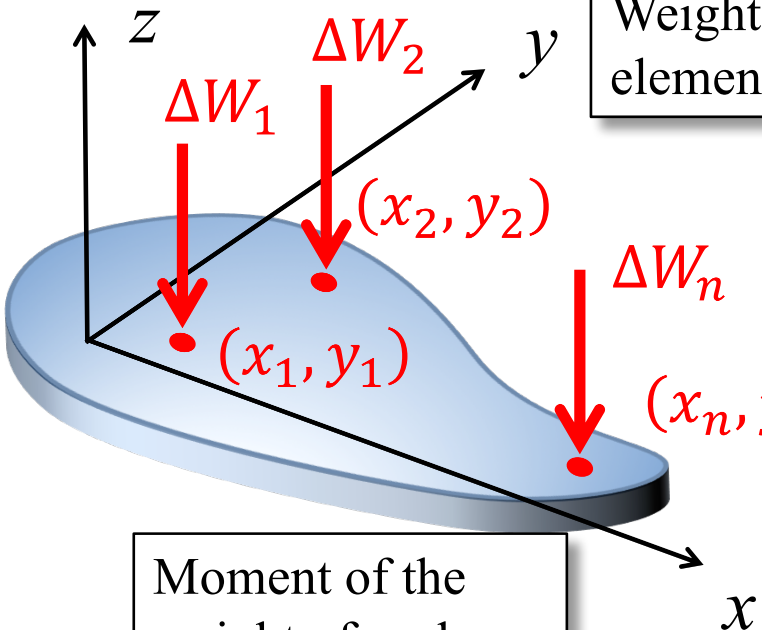
Center of Gravity



The golf club will balance at one point if it is held at its center of gravity.

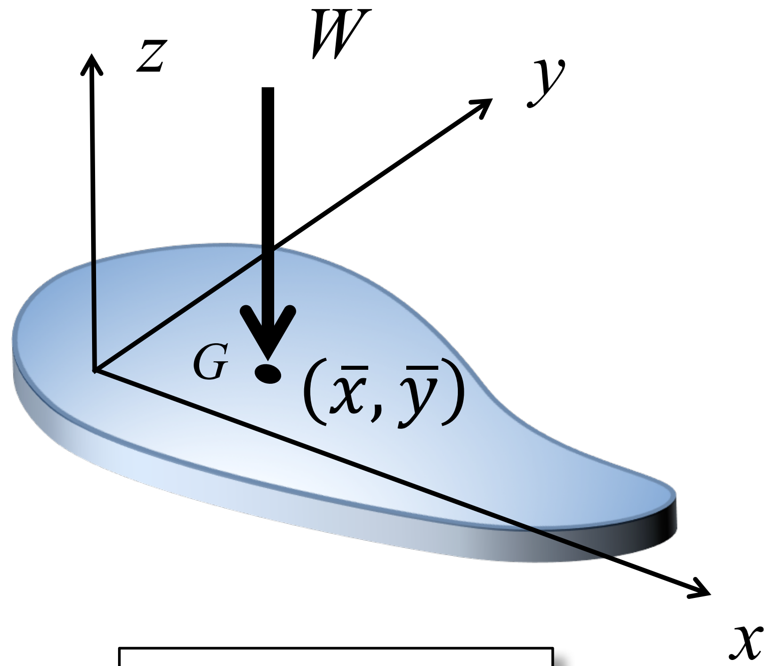
The center of gravity is the point where the resultant weight of the golf club acts.

Uniform Plate Divided into n Small Elements



Weight of each element = ΔW_i

=



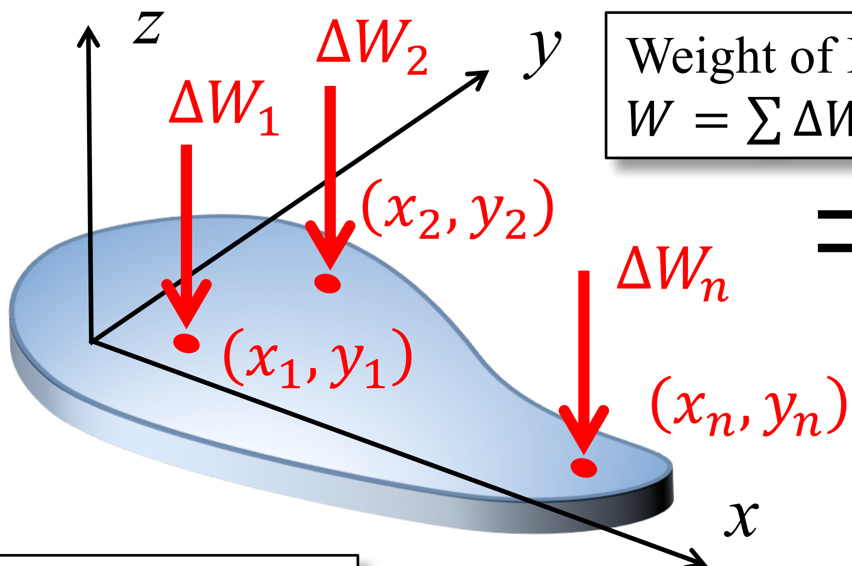
Moment of the weight of each element about the y axis = $(\Delta W_i)(x_i)$

Moment of the weight of each element about the x axis = $(\Delta W_i)(y_i)$

Moment of the entire plate about the y axis = $W\bar{x}$

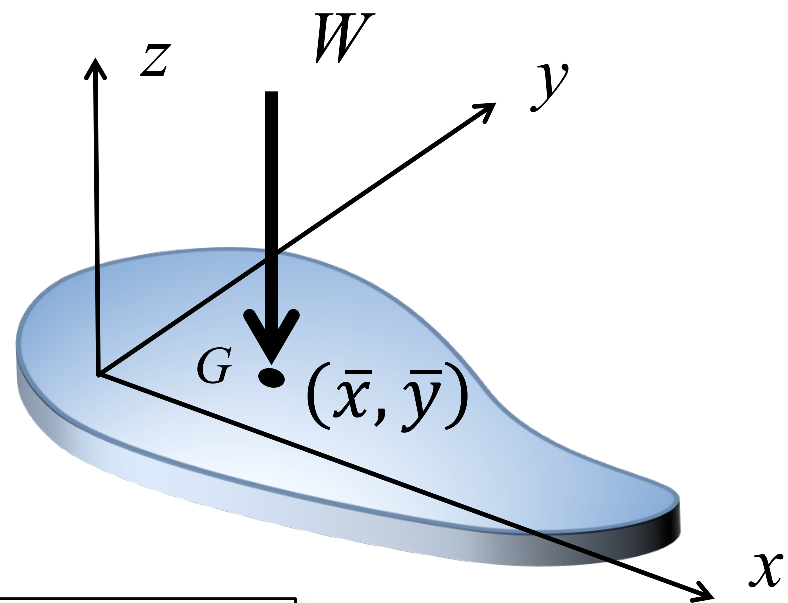
Moment of the entire plate about the x axis = $W\bar{y}$

Coordinates of the Center of Gravity



Weight of Plate
$W = \sum \Delta W_i$

$=$



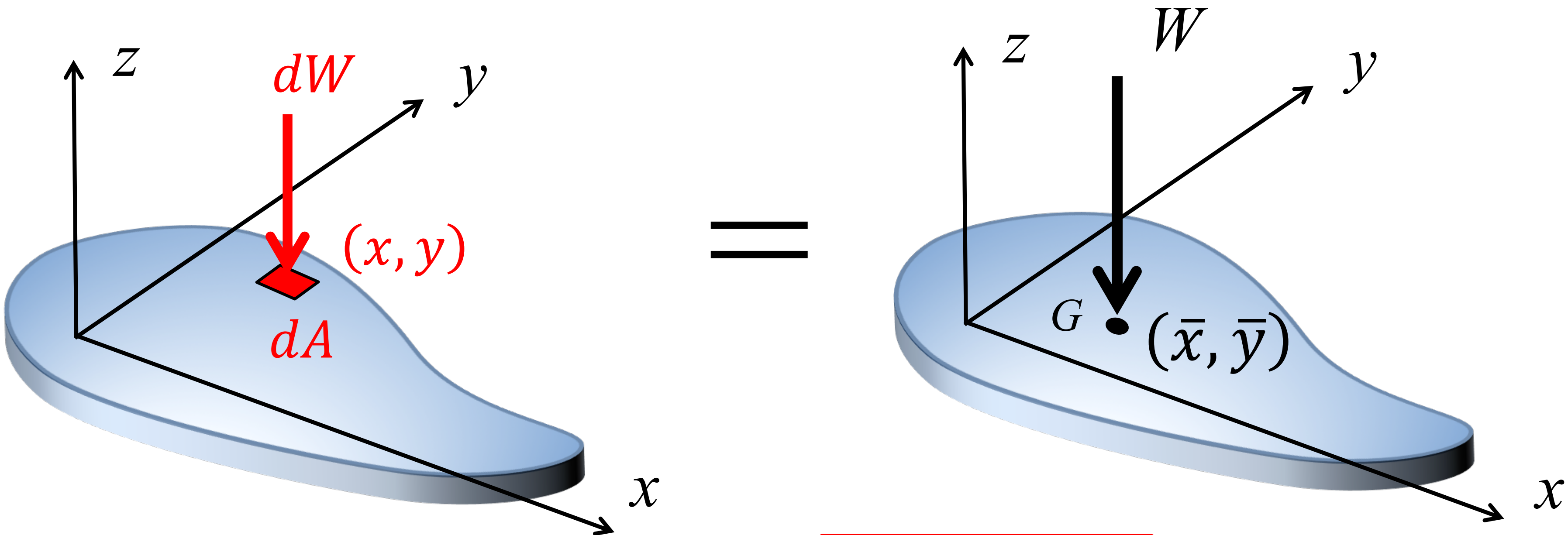
Moment of the weight of all elements about the y axis $= \sum \Delta W_i x_i$ $=$ Moment of the entire plate about the y axis $= W\bar{x}$

Moment of the weight of all elements about the x axis $= \sum \Delta W_i y_i$ $=$ Moment of the entire plate about the x axis $= W\bar{y}$

$$\bar{x} = \frac{\sum \Delta W_i x_i}{W}$$

$$\bar{y} = \frac{\sum \Delta W_i y_i}{W}$$

As the Number of Sections Gets Large

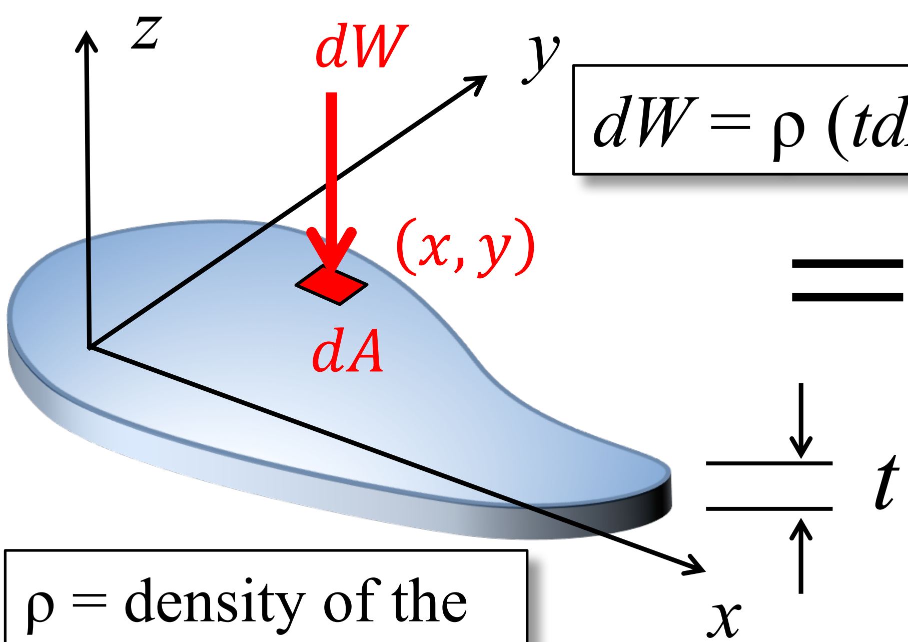


$$W = \iint dW$$

$$\bar{x} = \frac{\iint x dW}{W}$$

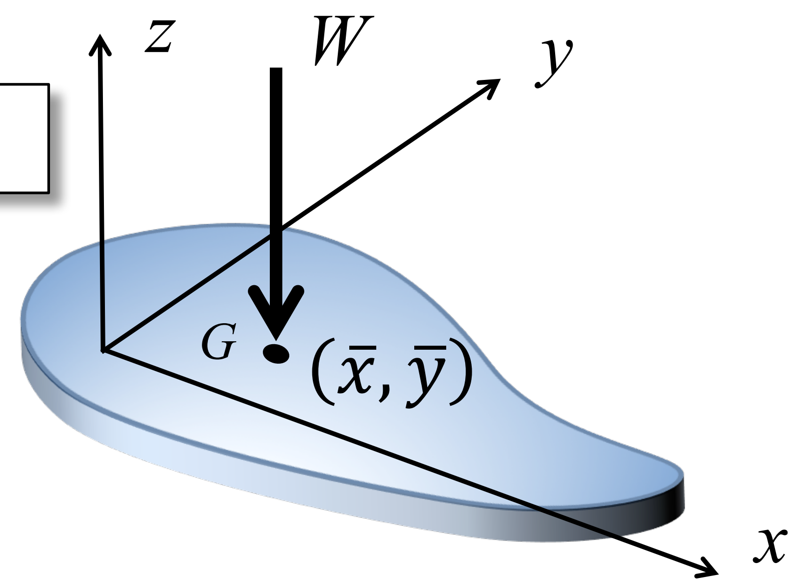
$$\bar{y} = \frac{\iint y dW}{W}$$

For a Body With Uniform Density the Center of Gravity Coincides with the Centroid of the Shape



$$dW = \rho\,(t\,dA)$$

$$=$$



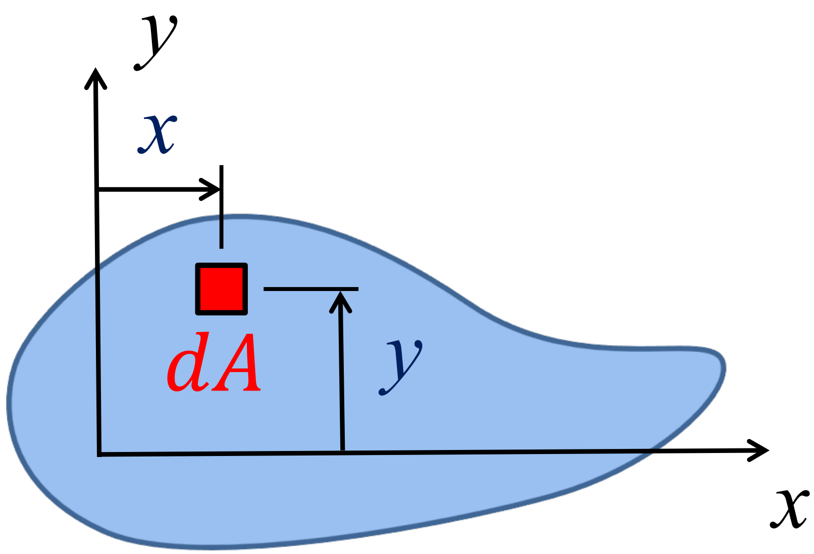
ρ = density of the uniform plate

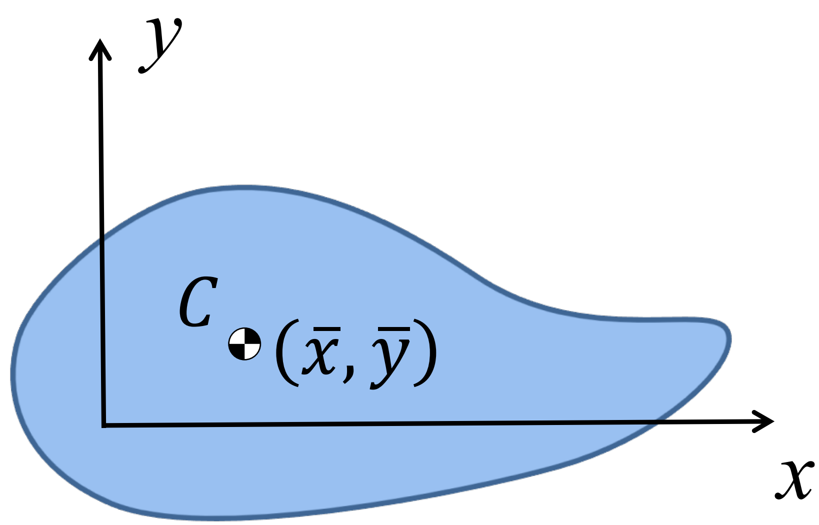
$$W = \rho t \iint dA = \rho t A$$

$$\bar{y} = \frac{\rho t \iint y\,dA}{W} = \frac{\iint y\,dA}{A}$$

$$\bar{x} = \frac{\rho t \iint x\,dA}{W} = \frac{\iint x\,dA}{A}$$

Centroid of an Area





$$A = \iint dA$$

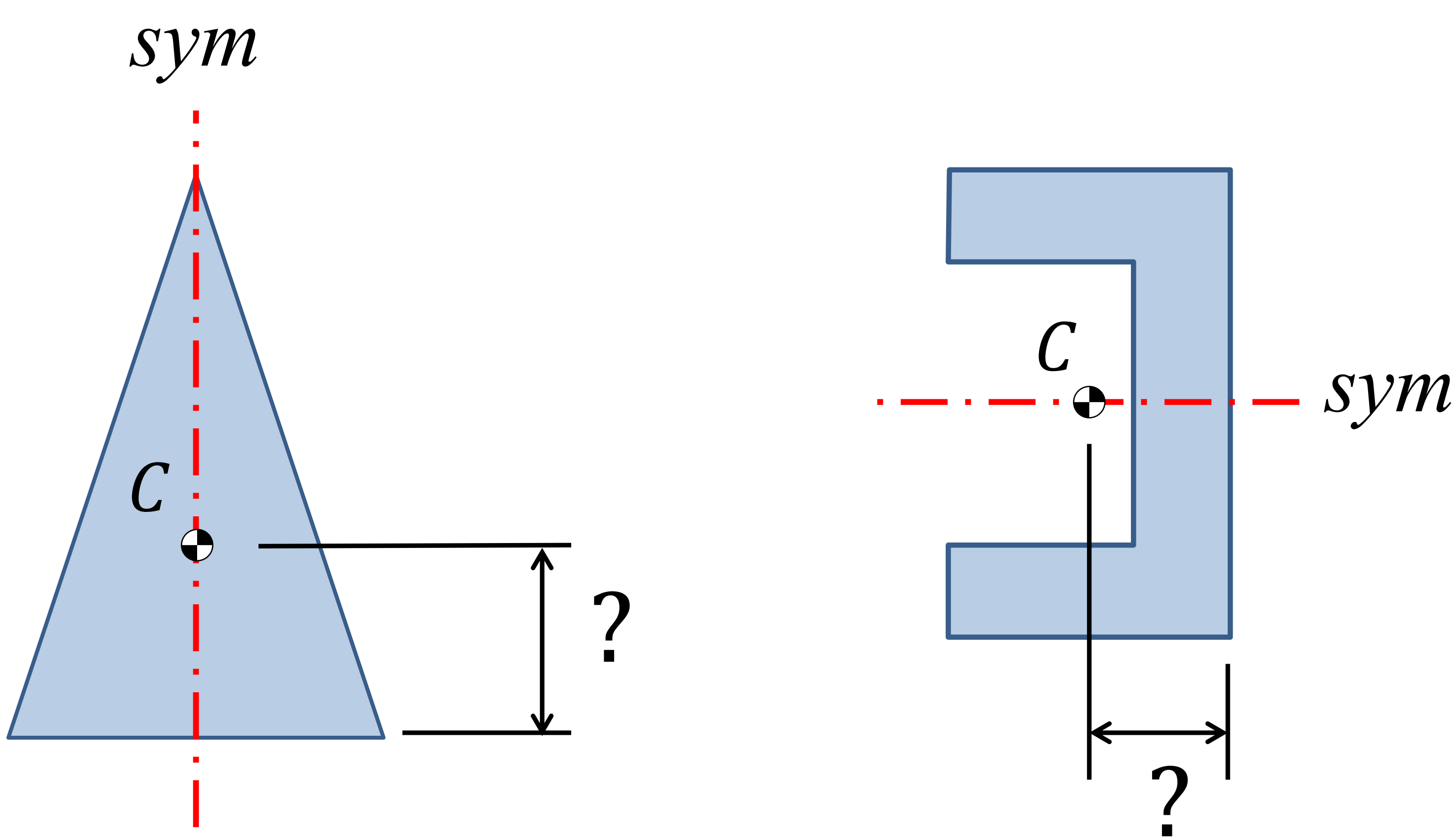
$$\bar{x} = \frac{\iint x dA}{A}$$

First moment of the area about the *y* axis

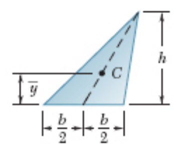
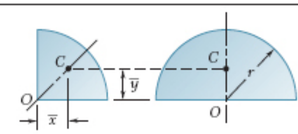
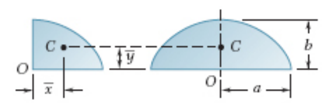
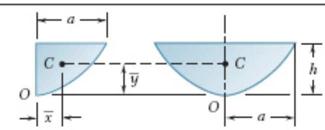
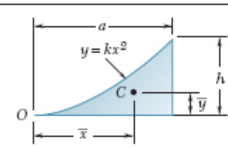
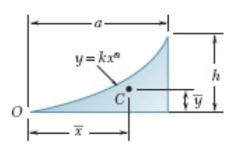
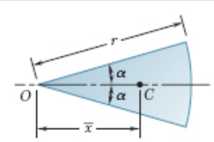
$$\bar{y} = \frac{\iint y dA}{A}$$

First moment of the area about the *x* axis

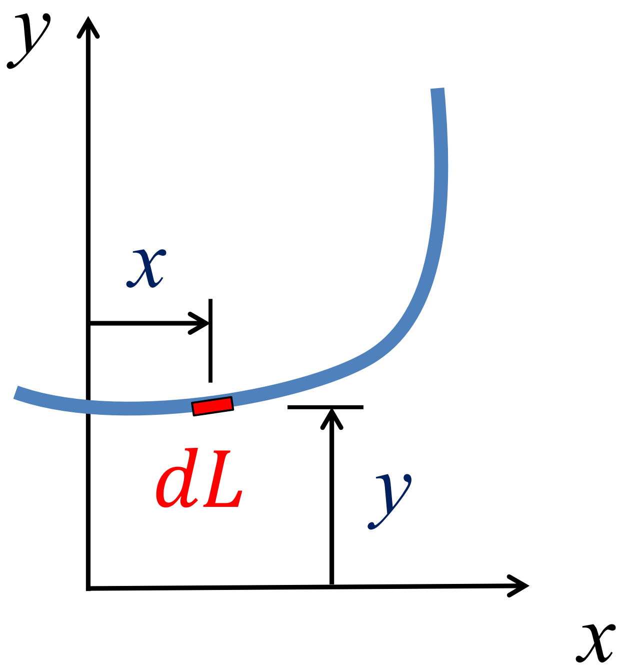
The Centroid of a Body Will Be Located on an Axis of Symmetry

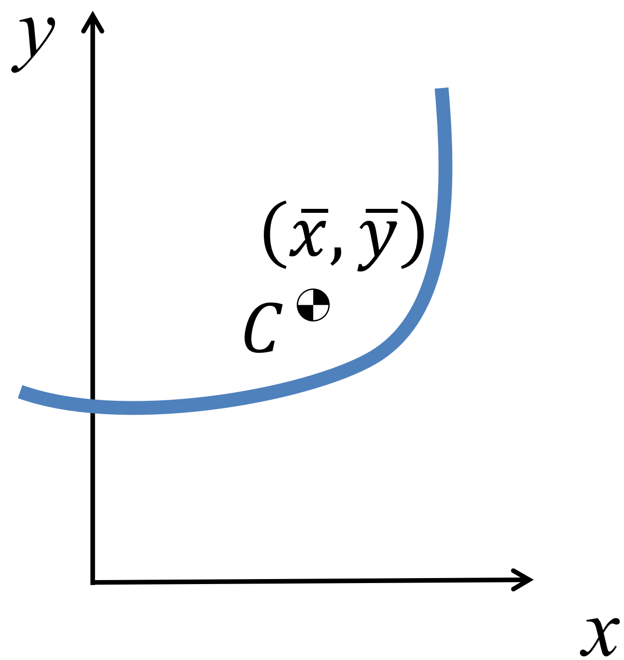


Tabulated Centroids of Common Areas Can be Found in the Textbook

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Centroid of a Line



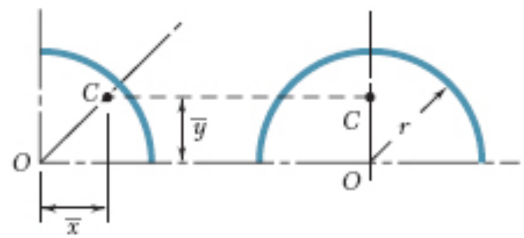
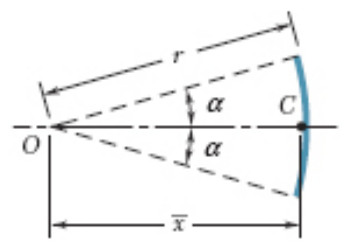


$$L = \iint dL$$

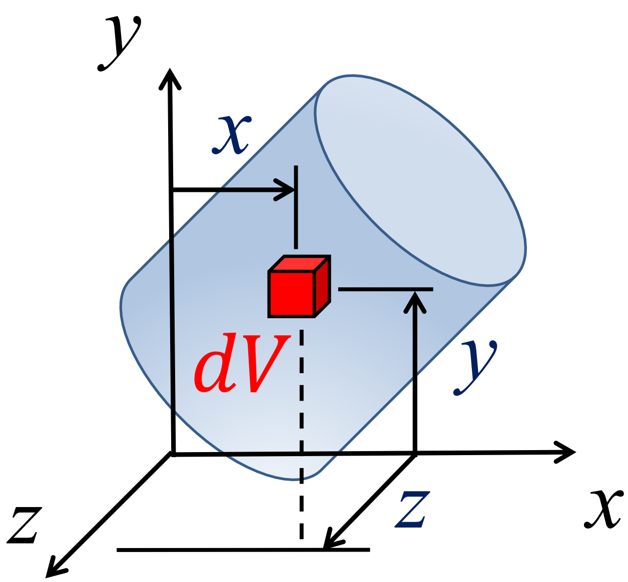
$$\bar{x} = \frac{\iint x dL}{L}$$

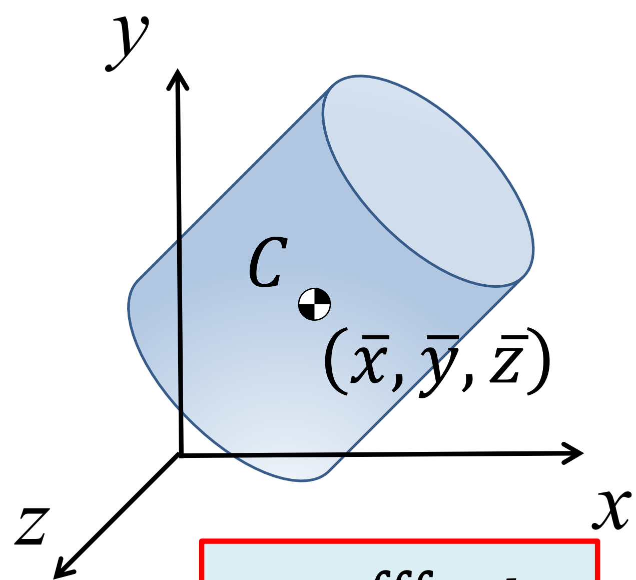
$$\bar{y} = \frac{\iint y dL}{L}$$

Tabulated Centroids of Common Lines Can be Found in the Textbook

Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Centroid of a Three-Dimensional Body





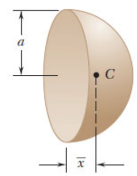
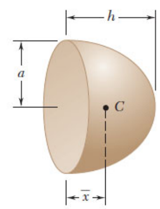
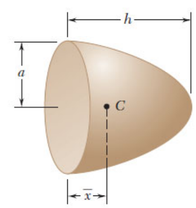
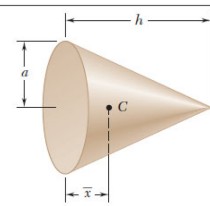
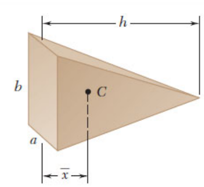
$$V = \iiint dV$$

$$\bar{x} = \frac{\iiint x dV}{V}$$

$$\bar{y} = \frac{\iiint y dV}{V}$$

$$\bar{z} = \frac{\iiint z dV}{V}$$

Tabulated Centroids of Common Three-Dimensional Bodies Can be Found in the Textbook

Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$
Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$
Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3}abh$